BT-MRE-I & II(S)-05.17-0681 Reg. No. [ ][ ][ ][ ][ ][ ][ ][ ]

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B. Tech. Degree I &II Semester Supplementary Examination in Marine Engineering May 2017

MRE 102 ENGINEERING MATHEMATICS II

(Prior to 2013 Scheme)

Time: 3 Hours Maximum Marks: 100

$(5 \times 20 = 100)$

I. (a) For the matrix $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{pmatrix}$, find non-singular matrices P and Q such that PAQ is in the normal form.

(b) Using matrix method, show that the equations
$3x+3y+2z=1,\ \ x+2y=4,\ \ 10y+3z=-2,\ \ 2x-3y-z=5$
are consistent and hence obtain the solution for x, y and z.

OR

II. (a) Prove that the function $\sin h\, z$ is analytic and find its derivative.

(b) Use Cauchy's integral formula to evaluate $\int_C \frac{e^{2z}}{(z+1)^4}\, dz$ where C is the circle $|z| = 2$.

(c) Expand $f(z) = \frac{z}{(z+1)(z+2)}$ about $z = -2$.

III. (a) Solve $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$.

(b) Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$ where λ is a parameter.

OR

IV. (a) Solve $(D^2 + D - 2)y = e^{-2x} + 2\sin h\, x$.

(b) Solve the following simultaneous equations
$\frac{dx}{dt} + 5x - 2y = t$; $\frac{dy}{dt} + 2x + y = 0$; given that $x = y = 0$ when $t = 0$.

V. (a) Obtain the Fourier Series for the function $f(x) = x^2, -\pi < x < \pi$.

Hence show that (i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{6}$

(ii) $\frac{1}{1^2} - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{12}$.

(b) Expand $\pi x - x^2$ in a half-range sine series in the interval $(0, \pi)$ upto the first three terms.

OR

VI. (a) Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$

as a Fourier integral.

(b) Prove that $\int_0^1 \frac{dx}{\sqrt{1-x^4}} = \frac{\sqrt{\pi}}{4} \frac{\Gamma(1/4)}{\Gamma(3/4)}$.

VII. (a) Find the Laplace transform of (i) $\sin^3 2t$ (ii) $t\ e^{-4t} \sin 3t$.

(b) Find the inverse Laplace transform of

(i) $\frac{4S+15}{16s^2-2s}$ (ii) $\frac{5s+3}{(s-1)(s^2+2s+5)}$

OR

VIII. (a) Apply convolution theorem to evaluate: $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$.

(b) Solve using Laplace transform:

$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 0$ where $y = 1, \frac{dy}{dt} = 2, \frac{d^2y}{dt^2} = 2$ at $t = 0$

IX. (a) A has two shares in a lottery in which there are 3 prizes and 5 blanks; B has three shares in a lottery in which there are 4 prizes and 6 blanks. Show that A's chance of success is to B's as 27 : 35.

(b) A random variable X has the following probability function:

values of X,	X :	0	1	2	3	4	5	6	7
	P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2$+k

(i) Find k (ii) Evaluate $P(X < 6)$ (iii) Find the minimum value of $x \in X$ such that $P(X \le x) > \frac{1}{2}$.

OR

X. (a) An urn contains 10 white and 3 black balls, while another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and there a ball is drawn from the latter. What is the probability that it is a white ball?

(b) Assume that on the average one telephone number out of fifteen called between 2 PM and 3 PM on week-days is busy. What is the probability that if 6 randomly selected telephone numbers are called (i) not more than three (ii) atleast three of them will be busy?

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